

# Influence of Laser Pulse Parameters on the Properties of $e^-e^+$ Plasmas Created from Vacuum

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We use the low density approximation within the kinetic theory approach to vacuum creation of an  $e^-e^+$  pair plasma (EPPP) in a strong laser field in order to investigate the dependence of the observed EPPP on the form and parameters of a single laser pulse. The EPPP distribution function is calculated for an arbitrary time dependence of the electric field in the multiphoton domain (adiabaticity parameter  $\gamma \gg 1$ ). The dependence on the field strength, the form and spectrum of the field pulse is investigated on the basis both analytical and numerical methods. The obtained results can be useful for examining some observable secondary processes associated with the dynamical Schwinger effect.

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## 1 Introduction

The short wavelength domain of the laser radiation is very prospective for experimental observation and investigation of the dynamical Schwinger effect [1–3]. The operating X-ray laser facilities [4, 5] are able to generate only very small electric fields in the focal spot with the amplitude  $E_0 \sim 10^{-3} - 10^{-2} E_c$ , where  $E_c = m^2/e$  is the Schwinger critical field strength. However, there are perspectives to achieve the Schwinger limit in this domain in the nearest future (e.g., [6, 7]).

In the present work we investigate the residual electron-positron pair plasma (EPPP) which is generated from vacuum after the cessation of a high-power laser pulse in the focal spot. In contrast to the well-known works [8, 9], our approach is based on the analysis of a kinetic equation (KE) [10, 11] that is an exact nonperturbative consequence of the basic equations of motion of QED for the case of a linearly polarized time dependent electric field ("laser field"). We limit ourselves to the domain of subcritical fields  $E_0 \ll E_c$  and to the case of rather short wavelength  $\lambda$  when the adiabaticity parameter  $\gamma = 2\pi E_c \lambda_c / E_0 \lambda$  is large,  $\gamma \gg 1$ . This corresponds to the multiphoton mechanism of the EPPP excitation ( $\lambda_c = 1/m$  is the Compton wavelength).

In problems of vacuum particle production, one usually studies the characteristics of the out-state, when the external field is switched off. This is due to the needs of an experiment aimed at finding free real particles. Recently, however, there is a growing interest in the properties of the intermediate states (mid-states), characterized by quasiparticle interactions in the presence of an external field. This is motivated by the likely role of the elementary reactions of the quasiparticles in the field and their possible experimental manifestations [1, 12–14].

The most complete description of the pair production process for both the mid- and the out-states is given by the momentum distribution function. Depending on the shape of the external field the vacuum excitations can form very complex and diverse momentum distributions which are essentially nonequilibrium ones. The complex form of the pair momentum spectrum should significantly affect the cross sections of elementary reactions

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and the development of avalanche-like electromagnetic cascades [13, 14]. It should also have some observable manifestations.

The most striking difference between the nonequilibrium spectrum of produced particles and the equilibrium one is a significant non-monotonicity of  $f(\mathbf{p})$ . This deformation is much more pronounced in the mid-state which is important for elementary reactions and cascades. This feature has been noticed in the works [1, 15] for the case of a periodic field, and was later studied more in detail for a Gaussian laser pulse in the work [16]. The representative oscillations of the momentum distribution with a scale set by the laser frequency received an explanation on the mathematical basis for the Stokes phenomenon in the theory of differential equations [17]. Other manifestations of such effects were studied in [18, 19].

It is important to note that these remarkable features of the momentum distribution function are not captured by WKB-like approaches [8, 9]. On the other hand, the only known exact solution for a homogeneous field in the form of a Sauter bell generates a smooth spectrum [20]. Apparently, for the occurrence of nonmonotonic deformations of  $f(\mathbf{p})$  different scales must be present in the spectrum of the external field. The actual laser field is required in the spectrum of two different time scales, so that the question of the form  $f(\mathbf{p})$  is essential for the experiment.

In the framework of using limitations, the following main results were obtained by both numerical and analytical approaches:

- the distribution function of the residual EPPP has a complicated quasiperiodical character in both the longitudinal ( $p_{\parallel}$ ) and the transversal ( $p_{\perp}$ ) momentum components resulting in a cellular structure in the momentum space;
- the profile function for switching-on (and switching-off) the laser signal has a considerable influence on the magnitude and structure of the momentum distribution of the residual EPPP;
- together with the usual multiphoton mechanism of the EPPP excitations the photon cluster mechanism of absorption of the laser field energy plays an essential role due to the relativistic nature of this system.

In Sect. 2 we explain the problem and give an analytic solution of the KE in the domain  $\gamma \gg 1$  and  $E_0 \ll E_c$ . Such analytical estimations are important due to the complications encountered in the numerical calculations. In Sect. 3 we present some numerical solutions of the KE in this domain. The basic results are summarized in Sect. 4.

## 2 Low density limit in the multiphoton domain ( $\gamma \gg 1, E_0 \ll E_c$ )

For the description of the EPPP vacuum creation we use the exact nonperturbative KE obtained in the work [10] for an arbitrary time dependent and spatially homogeneous electric field with linear polarization for which we use the Hamiltonian gauge  $A^{\mu}(t) = (0, 0, 0, A(t))$  and the field strength  $E(t) = -\dot{A}(t)$ ,

$$\dot{f}(\mathbf{p}, t) = \frac{1}{2} \lambda(\mathbf{p}, t) \int_{t_0}^t dt' \lambda(\mathbf{p}, t') [1 - 2f(\mathbf{p}, t')] \cos \theta(t, t'), \quad (1)$$

where

$$\lambda(\mathbf{p}, t) = eE(t)\varepsilon_{\perp}/\omega^2(\mathbf{p}, t) \quad (2)$$

is the amplitude of the EPPP excitations,  $\omega(\mathbf{p}, t) = \sqrt{\varepsilon_{\perp}^2(\mathbf{p}) + (p_{\parallel} - eA(t))^2}$  with the transverse energy  $\varepsilon_{\perp} = (m^2 + p_{\perp}^2)^{1/2}$  and the high frequency phase is

$$\theta(t, t') = 2 \int_{t'}^t d\tau \omega(\mathbf{p}, \tau). \quad (3)$$

The distribution function in the quasiparticle representation  $f(\mathbf{p}, t) = \langle in | a^+(\mathbf{p}, t) a(\mathbf{p}, t) | in \rangle$  is defined with the in-vacuum state  $| in \rangle$ . For the generalization of the KE (1) to arbitrary electric field polarization see [15, 21, 22].

In the low-density limit  $f \ll 1$ , the KE (1) leads to the formal solution

$$f_{\text{low}}(\mathbf{p}, t) = \frac{1}{4} \left| \int_{-\infty}^t dt' \lambda(\mathbf{p}, t') e^{i\theta(t, t')} \right|^2. \quad (4)$$

Here, the external field is switched on in the infinite past,  $t_0 \rightarrow -\infty$ . The approximation (4) implies that the external field is rather weak,  $E \ll E_c$ . Below we will restrict ourselves to the analysis of Eq. (4) in the limit  $t \rightarrow \infty$ , which defines the momentum distribution of the residual EPPP,

$$f_{\text{out}}(\mathbf{p}) = \lim_{t \rightarrow \infty} f_{\text{low}}(\mathbf{p}, t) = \frac{1}{4} \left| \int_{-\infty}^{\infty} dt \lambda(\mathbf{p}, t) e^{i\theta(t)} \right|^2. \quad (5)$$

Here the representation of the phase (3) via antiderivatives has been used,  $\theta(t, t') = \theta(t) - \theta(t')$ .

Hereafter we will consider the domain of the multiphoton mechanism of EPPP vacuum creation, where the adiabaticity parameter is large [8],  $\gamma \gg 1$ . A perturbation theory w.r.t. the small parameter  $1/\gamma \ll 1$  can be constructed here.

As a first step let us consider the function  $f_{\text{out}}(\mathbf{p})$  (5) in the leading approximation, when

$$\begin{aligned} \omega(\mathbf{p}, t) &\rightarrow \omega_0(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}; \\ \lambda(\mathbf{p}, t) &\rightarrow \lambda_0(\mathbf{p}) E(t), \quad \lambda_0(\mathbf{p}) = e\varepsilon_{\perp}/\omega_0^2(\mathbf{p}), \end{aligned} \quad (6)$$

and  $\theta(t) = 2\omega_0 \cdot t$ . This means that the contribution of the high frequency harmonics of the external field is omitted in Eq. (5). It leads to the simple result

$$f_{\text{out}}(\mathbf{p}) = \frac{1}{4} \lambda_0^2 \left| E(\omega = 2\omega_0) \right|^2, \quad (7)$$

where  $E(\omega)$  is the Fourier transform of the field strength  $E(t)$ . In this approximation the vacuum EPPP production takes place when for the frequency of the one photon  $e^-e^+$  pair creation process  $\omega_{1\gamma}$  holds  $\omega_{1\gamma} = 2\omega_0$ . This mechanism is exclusive here. Its intensity is regularized by the presence of the frequency  $\omega_{1\gamma}$  in the spectrum of an external field.

In order to facilitate EPPP creation, one can take into account the multiphoton mechanisms of EPPP production. These processes are described by the high frequency multiplier in Eq. (5). Let us use the non-perturbative method of photon counting assuming that the electric field is periodical,  $A(t) = A(t + 2\pi/\nu)$ , where  $\nu = 2\pi/T$  is the angular frequency and  $A(t) = A(-t)$ , in order to provide the property of being an even function of the quasiparticle energy,  $\omega(\mathbf{p}, t) = \omega(\mathbf{p}, -t)$ . The corresponding Fourier transform

$$\omega(\mathbf{p}, t) = \sum_{n=1}^{\infty} \Omega_n \cos n\nu t \quad (8)$$

leads to the decomposition of the phase in Eq. (5)

$$\theta(t) = 2\Omega_0 t + \sum_{n=1}^{\infty} a_n \sin n\nu t, \quad (9)$$

where  $a_n = 2\Omega_n/\nu n$ . In the case of a harmonic external field  $\Theta_0$  is the renormalized frequency [9]. Let us employ now the leading approximation  $\lambda(\mathbf{p}, t) \rightarrow \lambda_0(\mathbf{p}) E(t)$  (6) and a non-perturbative decomposition based on the well known formula

$$\exp(ia \sin \phi) = \sum_{n=-\infty}^{\infty} J_n(a) e^{in\phi}, \quad (10)$$

where  $J_n(a)$  is the Bessel function.

Let us now consider the integral in Eq. (5) and perform the substitutions (8)-(10), leading to

$$\begin{aligned} J &= \int_{-\infty}^{\infty} dt E(t) e^{i\theta(t)} \\ &= \int_{-\infty}^{\infty} dt E(t) e^{2i\Omega_0 t} \left\{ J_0(2\Omega_0) + \prod_{n=1}^{\infty} \sum_{k_n=1}^{\infty} J_{k_n}(a_n) e^{i(k_n n)\nu t} + (k_n \rightarrow -k_n) \right\}. \end{aligned} \quad (11)$$

The first term with  $J_0(2\Omega_0)$  describes the direct vacuum excitations at the frequency  $\omega = 2\Omega_0$  and corresponds to Eq. (7). If we are interested in excitations at lower frequencies, this contribution can be omitted.

A new feature of the multiphoton processes here is the cluster character of the energy absorption from the photon reservoir of the external field. The photon cluster of  $n^{\text{th}}$  order consists of  $n$  identical photons with the carrier frequency  $\nu$  and the total energy  $n\nu$ . As it can be seen from Eq. (11), it is possible that  $k$  of the  $n$ -photon clusters are absorbed simultaneously. The ordinary multiphoton process corresponds to the simplest "cluster" of the order  $k = 1$ . The appearance of photon clusters in the multiphoton processes in Eq. (11) is a consequence of the nonlinear field dependence of the quasiparticle energy  $\omega(\mathbf{p}, t)$ . The probability for the generation of an  $n$ -photon cluster is defined by the amplitude  $a_n$  in the decomposition (9) (the argument of the Bessel function  $J_k(a_n)$  in Eq. (11)) while the probability of the simultaneous  $k$ -cluster absorption corresponds to the Bessel function of the order  $k$ .

We rewrite the integral (11) in the following approximation:

$$J(n_{\text{max}}) = \int_{-\infty}^{\infty} dt E(t) e^{2i\Omega_0 t} \prod_{n=1}^{\infty} \sum_{k_n=1}^{\infty} \left\{ J_{k_n}(a_n) \exp\left(i \sum_{n=1}^{n_{\text{max}}} (nk_n)\nu t\right) + (k_n \rightarrow -k_n) \right\}, \quad (12)$$

where  $n_{\text{max}}$  is the maximal photon number in the cluster,  $J = J(n_{\text{max}} \rightarrow \infty)$ . Let us perform here the resummation procedure. In Eq. (12) we conserve the index  $k_n$  for notation of the  $k_n$  identical photon clusters where each cluster contains  $n$  photons. Then the total photon number in this photon set will be equal to  $N_k = nk_n$ . Let us replace now the summation over  $n$  by a summation over  $N_k$ . The integral (12) can then be rewritten as

$$J = 2\pi \sum_{k=1}^{\infty} E(\omega = 2\Omega_0 - \nu \sum_{N_k=1}^{N_{\text{max}}} [N_k]) \prod_{N_k=1}^{\infty} J_k(a_{N_k/k}). \quad (13)$$

Here  $N_k$  denotes an arbitrary integer multiple of  $k$  and  $N_{\text{max}} \sim n_{\text{max}}$  in Eq. (12).

Substituting Eq. (13) into Eq. (5), we obtain the generalization of Eq. (7)

$$f_{\text{out}}(\mathbf{p}) = \pi^2 \lambda_0^2 \left| \sum_{k=1}^{\infty} E(\omega = 2\Omega_0 - \nu \sum_{N_k=1}^{N_{\text{max}}} [N_k]) \prod_{N_k=1}^{\infty} J_k(a_{N_k/k}) \right|^2. \quad (14)$$

The summation is developed here by the order of the photon cluster. Below we will restrict the sum to the usual multiphoton processes ( $k = 1$ ).

Thus, the multiphoton mechanism leads to a lowering of the EPPP excitation frequency. The maximal effect is reached on the carrier frequency  $\nu$  of the external field, i.e. for the condition

$$2\Omega_0 - \nu \sum_{N_k=1}^{N_{\text{max}}} [N_k] = \nu. \quad (15)$$

Let  $\nu \ll m$ . Then the hypothesis of the phase randomization can be used,

$$\prec E(N\nu) E^*(N'\nu) \succ_{\Delta\omega} = |E(N\nu)|^2 \delta_{NN'}, \quad (16)$$

where  $N, N'$  are integers and  $\prec \cdots \succ_{\Delta\omega}$  denotes the averaging procedure over a small frequency interval  $\Delta\omega$ . The approximation (16) applied to Eq. (14) leads to the result

$$f_{\text{out}}(\mathbf{p}) = \pi^2 \lambda_0^2 \sum_{k=1} \left| E(2\Omega_0 - \nu \sum_{N_k=1}^{N_{\max}} [N_k]) \right|^2 \prod_{N_k=1} J_k^2(a_{N_k/k}). \quad (17)$$

In the case of the monochromatic external field Eq. (17) exhibits the accumulation effect. Using the formula  $\delta^2(\omega) = (T/2\pi) \cdot \delta(\omega)$  for  $T \gg 2\pi/\nu$ , one can obtain from Eq. (17) the EPPP production rate

$$I_{\text{out}}(\mathbf{p}) = \frac{1}{2} \pi E_0^2 \lambda_0^2 \sum_{k=1} \prod_{N_k=1} J_k^2(a_{N_k/k}) \delta(2\Omega_0 - \nu \sum_{N_k=1}^{N_{\max}} [N_k] - \nu). \quad (18)$$

Then the condition (15) defines the photon number required for the breakdown of the energy gap with the renormalized mass  $m_{\text{ren}} = \Omega_0(\mathbf{p} = 0)$ . Since the renormalized frequency  $\Omega_0(\mathbf{p})$  is a monotonously increasing function of  $p = |\mathbf{p}|$  with  $\min \Omega_0(\mathbf{p}) = m_{\text{ren}}$ , this photon number will be minimal for the particles created at rest,  $N_0 = N_{\max}(p = 0)$ . In order to investigate the momentum distribution of the EPPP it is necessary to consider a larger photon number,  $N_{\max} > N_0$  and  $N_{\max} \rightarrow \infty$  for  $p \rightarrow \infty$ .

The simplest situation corresponds to the usual multiphoton processes when the photon cluster mechanism is a minor,  $k = 1$ . The Eq. (18) in this case has the form

$$I_{\text{out}}(\mathbf{p}) = \frac{1}{2} \pi E_0^2 \lambda_0^2 \delta[2\Omega_0(\mathbf{p}) - \nu(N_{\max} + 1)] \prod_{n=N_0}^{N_{\max}} J_1^2(a_n). \quad (19)$$

The set of the Bessel function arguments  $a_n$  corresponds here to the spectral composition of the multiphoton process in Eqs. (8) and (9).

Already on the qualitative level it can be seen from Eqs. (18) and (19) that the momentum distribution of the residual EPPP has a cellular structure in momentum space stipulated by the presence of the Bessel function. It leads to some quasi-periodic behavior of  $f_{\text{out}}(\mathbf{p})$  w.r.t. the components  $p_{\parallel}$  and  $p_{\perp}$ .

Eqs. (18) and (19) allow to perform various estimations of EPPP characteristics (particle number density, momentum distributions) for different parameters of the laser radiation in the domain  $\gamma \gg 1$ .

As an example we find the estimate for the total intensity of EPPP production

$$I = 4 \int \frac{d^3 p}{(2\pi)^3} I_{\text{out}}(\mathbf{p}) \quad (20)$$

for  $\nu = m$  and  $E_0 = 0.1 E_c$ . It corresponds to the two-photon mechanism of EPPP creation. From Eq. (19) it follows then that the rate of EPPP production in the volume  $\lambda^3$  per period of the field action ( $E_0/E_c = \xi$ ) is

$$I = \frac{1}{2\pi} \xi^2 J_1^2(\xi^2/4) m^4 \approx 10^{-9} / (\lambda^3 T). \quad (21)$$

### 3 Numerical calculations

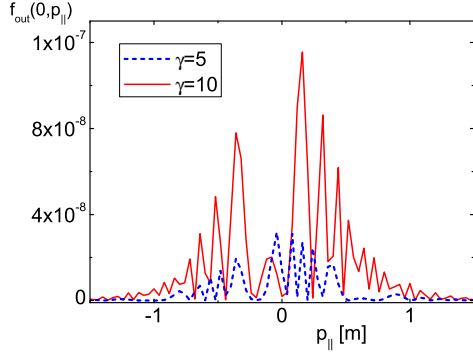
We explore field pulses with two types of shape: the radio pulse ( $N$  periods of the harmonic field)

$$E(t) = E_0 \sin \nu t, \quad 0 \leq t \leq N 2\pi/\nu, \quad N - \text{integer}, \quad (22)$$

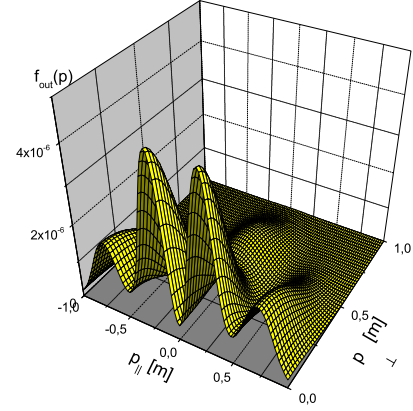
and the Gaussian pulse

$$E(t) = E_0 \exp(-t^2/2\tau^2) \cos \nu t, \quad \nu\tau = \sigma, \quad -10\tau \leq t \leq 10\tau. \quad (23)$$

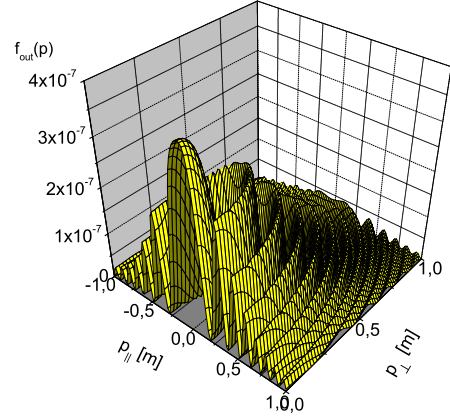
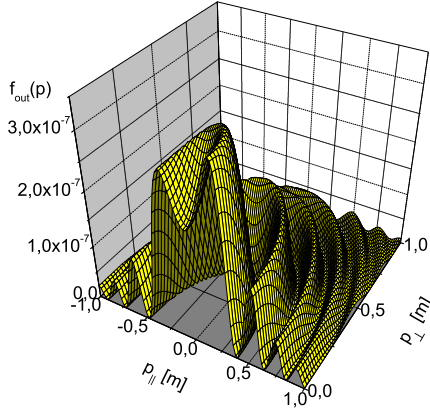
Figures 1, 3, 4 show the complicated behaviour of the momentum distribution  $f_{\text{out}}(\mathbf{p})$  for the periodical field (22) in dependence on the field parameters. This distribution turns out to be asymmetric w.r.t. the interchange



**Fig. 1** Section  $p_{\perp} = 0$  of  $f_{\text{out}}(\mathbf{p})$  for the pulse shape (22) for two values of  $\gamma$ .



**Fig. 2** The shape of  $f_{\text{out}}(\mathbf{p})$  for a Gaussian pulse (23) with  $\gamma = 10$ ,  $\sigma = 5$  and  $E_0 = 0.1 E_c$ .



**Fig. 3** Increasing complexity of  $f_{\text{out}}(\mathbf{p})$  in the field (22) with increasing  $N$  at  $E_0 = 0.01 E_c$  and  $\gamma = 24$ . Left panel:  $N=1$ ; right panel:  $N=2$ .

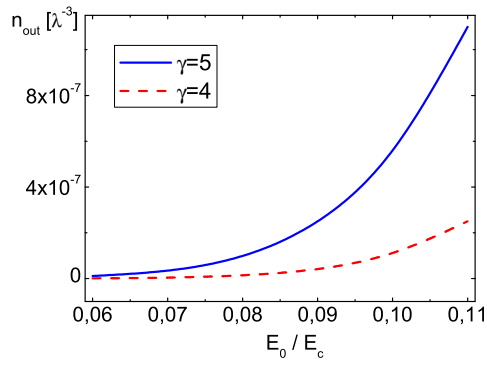
$p_{\parallel} \rightarrow -p_{\parallel}$  (Fig. 1). Increasing the number  $N$  of periods in the pulse (22) leads to a complication of the momentum distribution (Fig. 3) and to the generation of a cellular structure for large  $N \gg 1$  (Fig. 4). These details are lost when using the WKB methods [8, 9].

Analogous alterations can be observed in the case of the Gaussian pulse (23). For example, Fig. 2 shows the distribution function for  $\sigma = 5$  and  $\gamma = 10$ . Fig. 5 illustrates the strong dependence of  $n_{\text{out}}$  on the field strength. A comparison of the EPPP number density for the pulses (22) and (23) shows that the periodical field can produce a significantly higher output of observable pairs. The obtained estimates for  $n_{\text{out}}$  are notably different also from the well-known results of works based on the WKB approach.

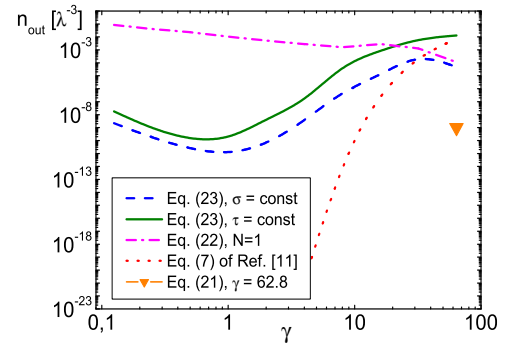




**Fig. 4** The complex cellular structure of the shape of  $f_{\text{out}}(\mathbf{p})$  at large  $N$  in a survey top view. Left panel: left  $N=50$ ; right panel:  $N=100$ .



**Fig. 5** Dependence of  $n_{\text{out}}$  on the field strength for a Gaussian pulse (23) for two values of  $\gamma$ .



**Fig. 6** Comparison of  $n_{\text{out}}$  for different pulse shapes and different theoretical approaches.

## 4 Conclusions

In the present work we have done first steps in the investigation of the residual EPPP based on a kinetic theory foundation for strong nonperturbative QED. We restricted ourselves here to the multiphoton domain  $\gamma \gg 1$  and the low-density approximation. We have shown that the momentum distribution of the residual EPPP is a strong nonequilibrium one and has a complicated cellular structure in the momentum space depending on the characteristics of the laser radiation. As a result of the numerical analysis it was observed that the residual EPPP density depends strongly on the pulse shape for switching on (and switching off) the laser field. Finally, a theoretical approach was developed for estimating different characteristics of the residual EPPP. This method allows to take into account both the usual multiphoton processes of EPPP excitation and also the new photon cluster mechanism.

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